



Statistical Treatment of the Gamma-Ray Spectrum Using the Language Programming R

Inaam H. Khadim^{1*}, Khalid Hussain Hatif²

Abstract

In this work we use the laws of statistics and statistical distributions software to try to understand the random statistical nature of the dissolution of gamma rays and their interaction with the material by studying the spectrum regions of the sodium-22 spectrum using the statistical programming language R. such Normality test, Anova Test as well as Pearson correlation coefficient test and the linear Regression test and we were found that there is no strong statistical relationship between the random variables studied using linear regression, also it was found that the average mean of the standard samples follows the normal distribution and this was confirmed by using the T-test, there is also a slight difference between the sampling random mean and the standard mean for the same samples using the same statistical distribution.

Key Words: Gamma Ray, Scintillation Detector, Statistical Distribution, Language R, Anova Test, Pearson Correlation.

DOI Number: 10.14704/nq.2021.19.2.NQ21017

NeuroQuantology 2021; 19(2):50-60

50

Introduction

Radioactive decay is a random process, in addition as the interaction of a gamma ray with matter is follow the laws of probability, that is, under certain conditions such as the spectrum of gamma ray photons or the atomic number with which the interaction occurs [1], the possibilities are different for the photoelectric effect, the Compton scattering, or the pair production, so thus subject to the laws Statistics [2]. So, the radiation emitted from nuclear decay is subject to a certain degree of statistical variability. These inherent fluctuations represent an inevitable source of uncertainty in all nuclear measurements and often they can be the predominant source of inaccuracy or error So we used programming and statistical tests to study the spectrum of the sodium -22 source. We analyzed the spectrum areas from the total area of the spectrum, photopeak, and others by using the statistical programming language R.[3] The R statistical program is one of the most popular statistical

programs. This program language is similar to the S programming language that was developed in Bell Lab[4].

R Studio is an operating environment for r statistical software with a range of wonderfully designed windows and tools that help manage the programming session in a structured manner that helps increase productivity and creativity. The use of such a program does not require a different programming language than R [5].

Experimental Methods

An electronic counting and analysis system was used by using NaI (Tl) sodium iodide crystal detector with size (3 "x 3") cm was measured based on the high penetration strength of the gamma ray in the materials by, the equipped by a company (Spectrum Techniques LLC), the nuclear measurements and analysis were done a computer program called UCS_30.

Corresponding author: Inaam H. Khadim

Address: ^{1*}Department of Physics, Collage of Science, University of Babylon, Iraq; ²Department of Physics, Collage of Science, University of Babylon, Iraq.

^{1*}E-mail: stud.inaam.hadi@uobabylon.edu.iq

Relevant conflicts of interest/financial disclosures: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

Received: 06 January 2021 **Accepted:** 18 February 2021



The UCS30 ADVANCED Spectrometer System contain 4096 channel MCA with internal preamplifier, high voltage (0-2048V), upper and lower-level discriminators and multichannel scaling for half-life and decay studied. [6]. Figure 1: shows Nuclear counting system NaI(Tl) in present study radiation detector (NaI(Tl) scintillation counter) is a pulses of light produced in a transparent material by the passage of a particle [7].

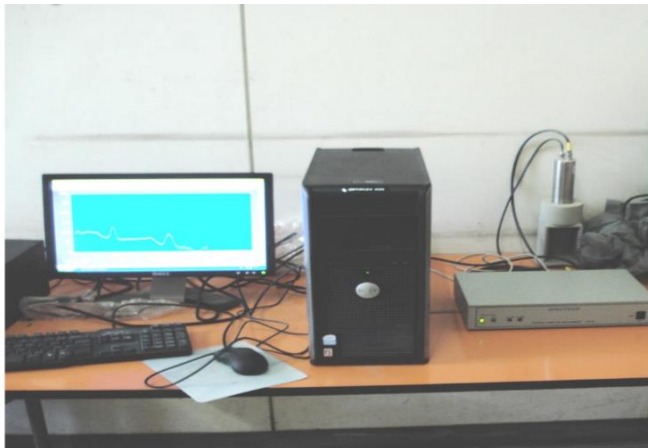


Figure 1. Nuclear detection system: NaI(Tl)

The iodine provides most of the stopping power in sodium iodide. These crystalline scintillators are characterized by high density, high atomic number, its efficiency and the high precision and counting rates are a consequence of the extremely short duration of the light flashes, from about 10^{-9} to 10^{-6} sec, that are possible and pulse decay times of

approximately (1 μ sec). Scintillation exhibit high efficiency for detection of gamma rays and are capable of handling high count rates. [8].

In general, a scintillation detector consists of:

Scintillator. A scintillator generates photons in response to incident radiation.

Photodetector. A sensitive photodetector which converts the light to electrical signal and electronics to process this signal. [9].

scintillation counters can be used to determine the energy, as well as the number, of the exciting particles (or gamma photons). For gamma spectrometry, the most common detectors include sodium iodide (NaI) scintillation counters and high-purity germanium detectors. [10].

Most scintillation detectors are only sensitive to one type of radiation. So NaI detectors will pick up gamma radiation, but not alpha or beta. every time a gamma hits the crystal it the NaI detectors for is identifying specific radionuclides by measuring the energy of each individual gamma that enters the crystal this process is called gamma spectroscopy, or can also be called multi-channel analysis (MCA).

The basic principle behind gamma spectroscopy is that every gamma-emitting radionuclide emits a gamma ray (or a few gammas) with very specific energies and if we can identify the gamma energies precisely enough then we can identify the radionuclide(s) present. so, sodium-22 (^{22}Na) gives off a gamma peak at 1.274 (MeV) as Figure (2). [11]

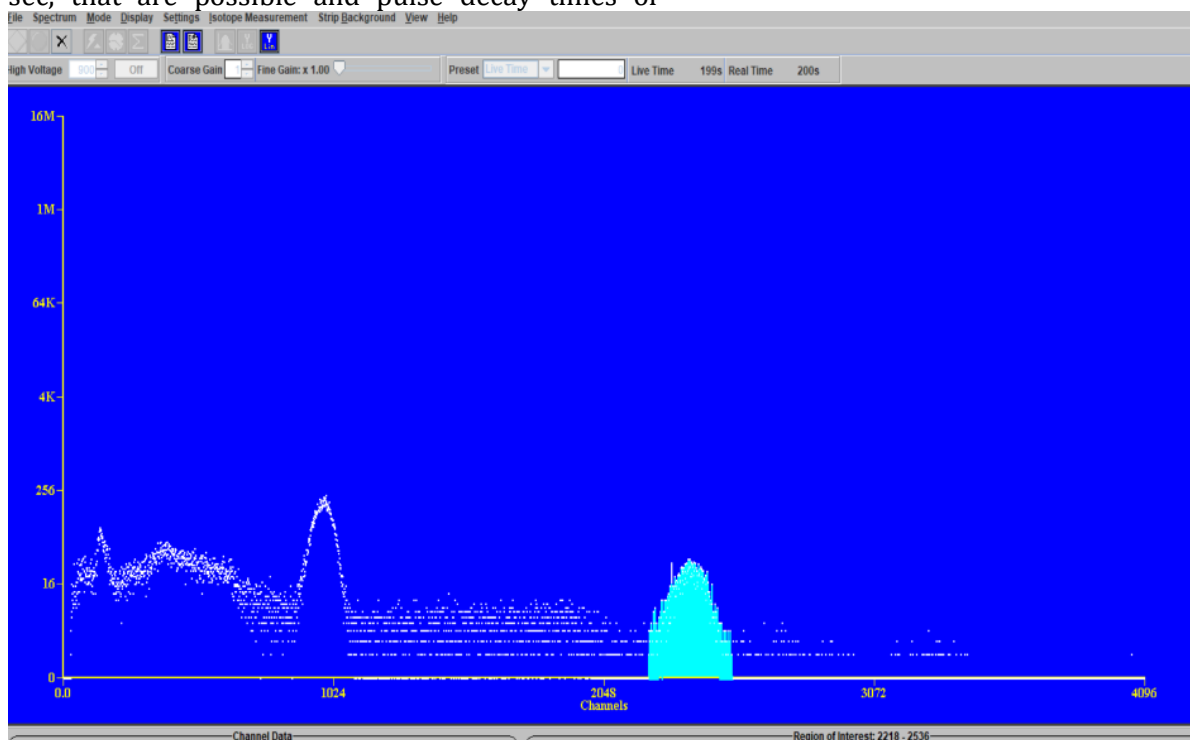


Figure 2. Spectrum Sodium-22.



The spectrum produced by the emission of these photons has a single photopeak. Two hundred readings gathered. The sodium spectrum and the collection time of each spectrum is (200 sec) with the voltage 900 volt and 4096 channel and the distance is (7cm) between the radioactive source and the detector.

Theoretical Part

Random variable is a variable that represents the numerical results of a random phenomenon. The random variable must be measurable. There are two types of random variables:

First: (discrete random variable) such as binomial distribution. It can be described as a probability mass function that is represented by a function[12]

$$F(X)=P(X=X) \tag{1}$$

Where it represents the probability of taking the random variable X Value with x. Its characteristics include:

- a) $F(X_i) \geq 0$, for $i=1, \dots, n$.
- b) $\sum N_i = 1$ $F(X_i) = 1$ $\tag{2}$

Second: (continuous random variable) such as Normal distribution and Exponential distribution. It can be described as a probability density function that is represented by a function.[11]

$$f(x) = P(X=x) \tag{3}$$

Where it represents the probability of taking the random variable X Value with x. Its characteristics include:

- a) $f(X_i) \geq 0$, for $i=1, \dots, n$.
- b) $\int_{-\infty}^{\infty} f(x) \dots \dots \dots \tag{4}$

Single Contrast Test (One-Way ANOVA Test) in R-R Programming Language

One-Way ANOVA Test is a statistical method used to show the difference between two or more averages by analyzing the difference within and between different categories. It is mathematically described as follows[13] :

$$H_0 = X \sim N(\mu, \sigma^2) \text{ or } H_1 = X \neq N(\mu, \sigma^2) \tag{5}$$

Also

$$H_0: \sigma_1^2 = \sigma_2^2 = \dots = \sigma_r^2 \text{ or } H_1: \sigma_i^2 \neq \sigma_j^2 \text{ for some } \tag{6}$$

Linear Decline using R Programming Language

Linear regression is a statistical method that aims to create a linear relationship between dependent variable and independent variables, as well as to predict (prediction) of the values of the dependent variable. The linear decline can be defined mathematically as follows[14] :

$$y = \alpha + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p + \epsilon \tag{7}$$

where it's called y dependent variable, response variable, outcome variable, also called x_1, x_2, \dots, x_p Independent variables, explanatory variables, or predictor variables. Residuals are defined as the difference between the observed (actual) value of the child variable and the child variable value that is predicted using the regression line.[15]

Results

In this paper, the radioactive nature of the ²²Na source was studied spectrum areas where the net total area was measured in addition to the two net photopeak spectrum of the sodium-22 as shown in Table.

Table 1. 22Na spectrum

sample	Total Area		Photopeak 1		Photopeak 2	
	Net	Cross	Net	Cross	Net	Cross
1	41,281	43,228	13,392	15,274	2,697	3,395
2	36,893	43,694	14,100	15,519	2,957	3,446
3	35,222	43,149	12,689	15,120	2,833	3,437
4	31,263	43,333	14,087	15,461	3,030	3,402
5	35,567	43,318	13,533	15,274	2,993	3,505
6	31,188	43,348	13,923	15,140	3,136	3,412
7	39,330	43,396	13,904	15,369	2,930	3,468
8	36,314	43,335	13,800	15,511	2,802	3,436
9	40,211	43,640	13,662	15,202	2,871	3,384
10	38,150	43,338	13,430	15,321	2,881	3,406
11	29,105	43,083	13,826	15,200	3,108	3,511
12	33,924	43,338	13,760	15,248	3,264	3,548
13	33,287	42,933	13,840	15,353	2,626	3,222
14	31,191	43,279	13,697	15,291	2,673	3,489
15	35,869	43,444	14,064	15,367	2,803	3,555
16	25,856	43,515	13,807	15,272	2,911	3,437
17	41,610	43,519	13,812	15,343	2,768	3,439
18	37,426	43,241	13,747	15,380	2,452	3,454
19	30,207	43,669	13,767	15,400	3,208	3,647
20	41,262	42,965	13,940	14,964	2,727	3,376
21	31,502	43,680	13,547	15,318	2,578	3,518
22	27,873	42,163	13,540	14,957	2,783	3,268

sample	Total Area		Photopeak 1		Photopeak 2	
	Net	Cross	Net	Cross	Net	Cross
101	39,431	43,362	14,608	15,460	2,936	3,520
102	31,208	43,340	13,730	15,312	3,310	3,604
103	41,732	43,427	14,128	15,494	2,844	3,590
104	32,574	43,009	13,840	15,120	3,183	3,499
105	41,449	43,395	13,741	15,349	2,850	3,476
106	34,838	43,349	14,160	15,484	2,931	3,486
107	27,434	43,608	14,689	15,436	3,091	3,571
108	39,549	43,174	13,829	15,423	2,899	3,431
109	41,361	43,260	13,783	15,224	2,904	3,453
110	26,069	43,145	13,519	15,140	2,943	3,535
111	41,388	43,215	13,919	15,476	2,921	3,459
112	31,654	43,022	13,772	15,233	2,979	3,382
113	31,542	43,620	13,503	15,451	3,291	3,581
114	36,385	43,066	13,296	15,007	2,760	3,431
115	22,593	43,351	13,642	15,328	3,283	3,561
116	32,999	43,155	13,538	15,132	2,889	3,378
117	32,967	43,234	14,048	15,165	3,079	3,469
118	32,799	43,243	13,939	15,290	3,255	3,538
119	35,175	43,283	13,623	15,207	2,889	3,540
120	33,408	43,151	13,689	15,178	3,153	3,441
121	41,432	43,341	14,096	15,248	2,500	3,366
122	33,566	42,770	12,901	14,801	3,082	3,511



Table 1 shows the spectrum areas of the radioactive sodium source, where 200 spectra were taken at a time of 300 seconds, an operating voltage of 900 volts, and amplification 1. The table was divided into two parts, each section containing 100 spectra in order to reduce the size of the table.

where: Net is indicates to the net area peak, This table was abbreviated in Table 2.

Table 2. summary spectrum 22Na

summary	Total.Net	Photo.1Net	Photo2Net
Min	22589	12689	2.774
Max	42000	18389	3399.000
Mean	34706	13823	2946.144
Median	34524	13782	2957.000
1st Qu	31653	13579	2810.500
3rd Qu	38559	14027	3130.000
IQR	6906	448	319.5
μ	34705.78	13823.34	2946.144
Var	21294226	231988.6	92795.14
σ	4614.567	481.652	304.6229
Mean (sam.)	34566.26	13837.41	2945.475
Sd (s)	4644.406	503.0848	321.2496
X.bar	34693	13824.54	2945.777
sd (bar)	143.9067	14.47532	9.869021
range	22589 42000	12689 18389	2.774 3399.000

Table.2 shows a summary for sodium spectrum, where : **Total.Net** refer to the net total area of the spectrum, **Photo.1Net** and **Photo.2Net** refers to the Net area for photopeak, and min and Max are refers to values sample in small and large value respectively, also **IQR** is the inter-quartile **range** which is the distance between the third quartile an account of the first **1st Qu** quarter and the third quarter **3rd Qu** of the data and μ refer to mean for population value, var is the variance to sample, σ is the variance to population, **X.bar** is standard mean, **sd** is the standard deviation and, as Figures from (3-1).

Statistical Analysis

The Figures 3 and 4 gives a summary of the statistical distributions on the studied environment samples, where is Figures 3 a histogram that shows the graphical relationship between the frequency of the sample and the mean of the spectrum. and the Figure 4 is the boxplot of total Area of spectrum and Photopeak Area.

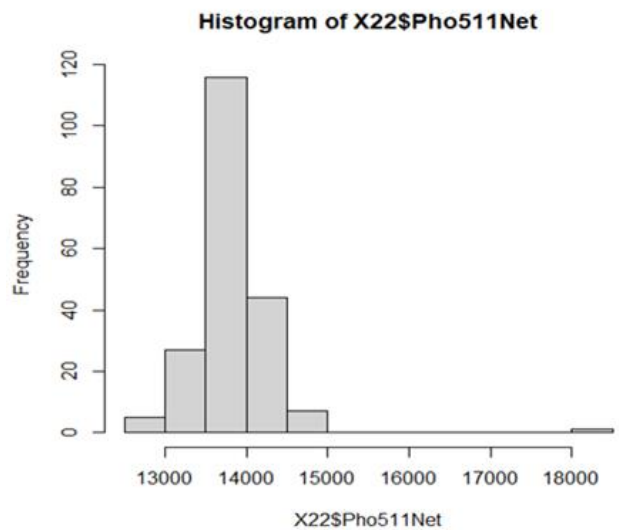
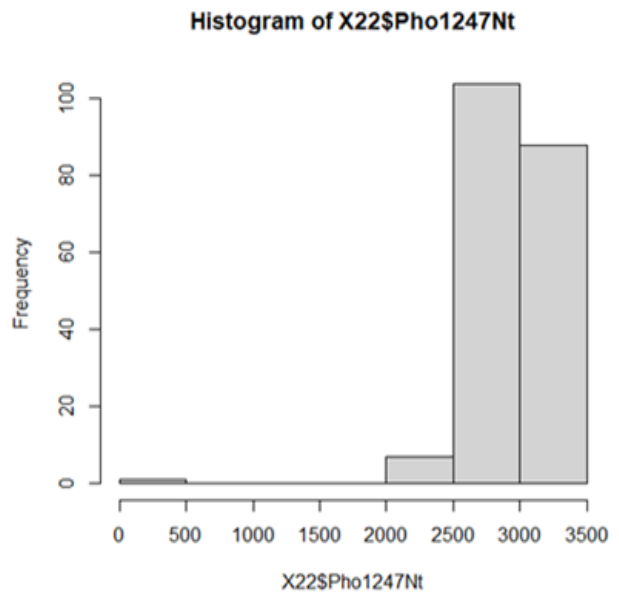
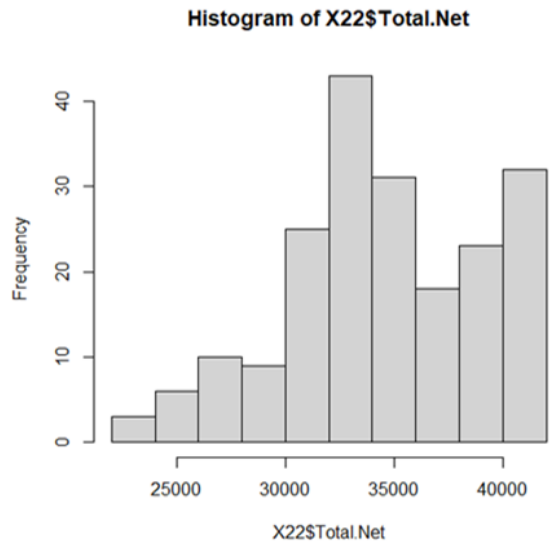


Figure 3. A- Histogram between the data (frequency) Total Net Area
 B- Histogram between the data Photopeak 1 Net Area
 C- Histogram between the data Photopeak 2 Net Area



The histogram A represents the net area of the total spectrum on the x-axis, and the data on the y-axis, where we notice its difference from figures B and C, which shows the net area of the two photopeaks of the sodium source, where figure C has fewer columns compared to figures A and B, we can the histogram is based on estimating the distribution and spread of the data around the mean, and we note the division of the x-axis with equal periods, and the height of each square represents the number of observations that fall within the mean.

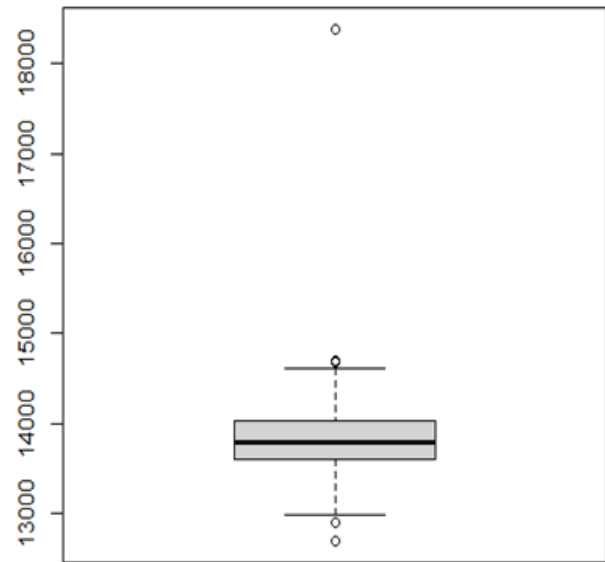
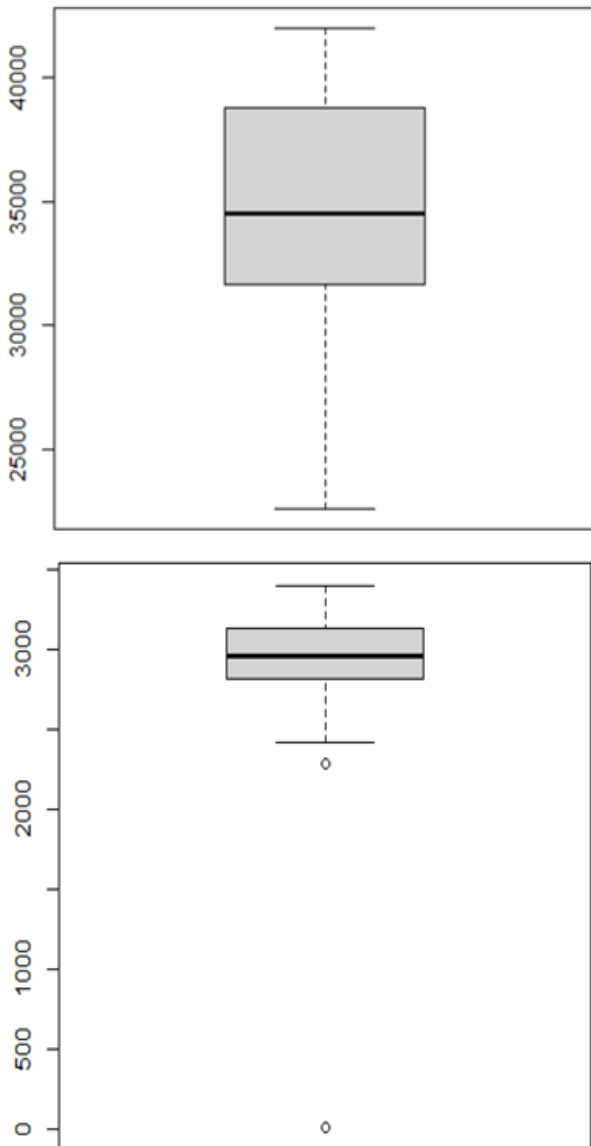


Figure 4. A- boxplot of Total Area of spectrum and Photopeak Area
 B- boxplot of Photopeak 1 Net Area
 C- boxplot of Photopeak 2 Net Area

Figure 4 displays the diagram, which provides a clear and comprehensive picture of the distribution of data and consists of five values: the smallest value, the first quartile, the third quartile, and the largest value. Where about 50% of the data are collected in the box, as for the rest of the values, they are distributed between the lowest and the highest value, and the broad black line represents the median (the second quarter), and we notice a clear difference between the three shapes and this is evidence of the difference in the spread of data around the mean.

To find out the homogeneity of the variance we use Normality Test or the Kolmogorov-Smirnov test, shapiro. test, and Q-Q. norm for the data samples as table 3.

Table 3. Test of Normality

Type of test	Tota.Net			Phot1Net			Phot2Net		
	X ₂₁	X _{21.s} amp	X _{21.} bar	X ₂₂	X _{22.s} amp	X _{22.} bar	X ₂₃	X _{23.s} amp	X _{23.} bar
Kolmogorov-Smirnov test	2.2e-16	0.1045	0.8528	2.2e-16	0.0092	0.239	2.2e-16	0.0562	0.0743
shapiro.test	4.62e-05	4.627e-05	0.4518	2.2e-16	2.2e-16	0.0023	2.2e-16	2.2e-16	5.954e-05



Table 3 shows the normality test that shows whether the data follows to a normal distribution or not, so we note p.value if it is greater than (0.05) it is obey to the normal distribution (or take H0), and if it is less than 0.05, it is not obey to the normal distribution (or take H1), as shown in (Eq.5).

where (X21, X22, X23) refers to data in population, (X21.samp, X22.samp, X23.samp) refer to data at random sampling and X21.bar, X22.bar, X23.bar) indicate to standard sample data.

In Shapiro's test, we note that all data are not subject to a normal distribution depending on the p. value. except for the standard samples at Tota.Net and Phot1Net, we notice that it differs from the rest of the data in that it is obey to a normal distribution. because it is standard sampling.

Q.Q. Normal Test

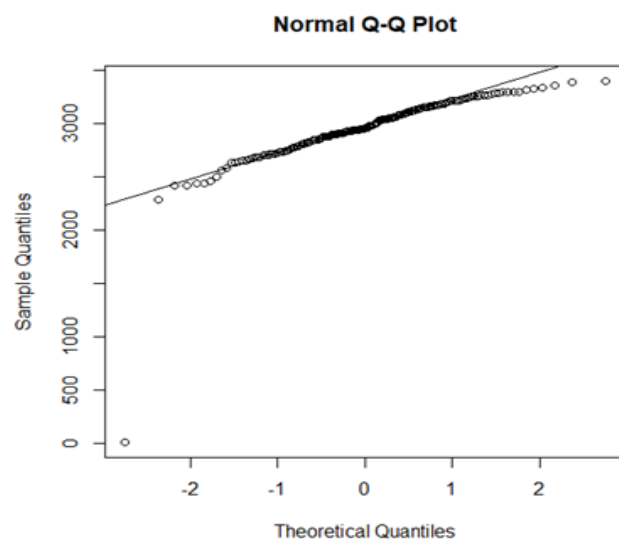
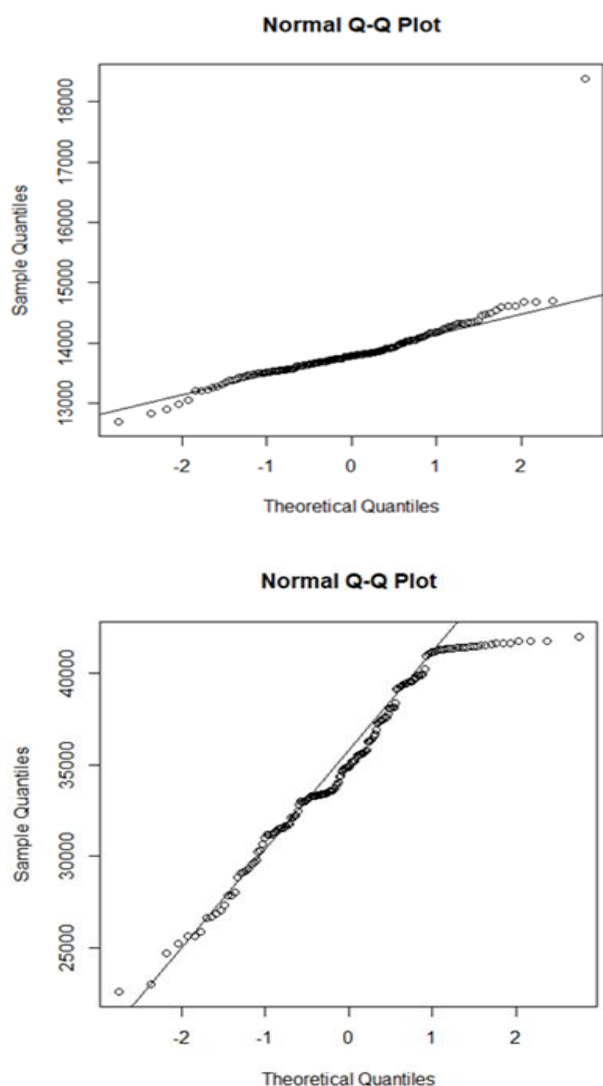


Figure 5. The figure shows the spread of the data
 Total Area of spectrum
 Photopeak 1 Net Area
 Photopeak 2 Net Area

In figure 5, we notice that if the points are applied along a straight line, the data are homogeneous and follow to a normal distribution.

To study the homogeneity of the data, we use Eq(5).

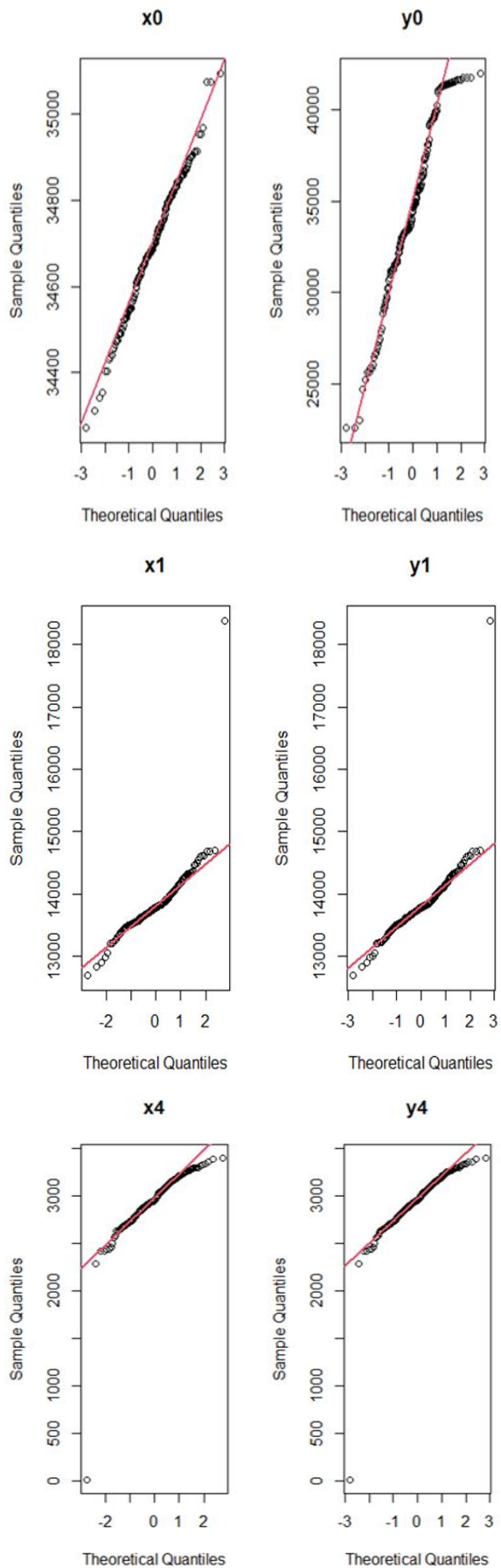
Table 4. Homogeneity of variances

Type of test		Tota.N et	Phot1Net	Phot2Net
bartlett.test		0.7281	0.187	0.8615
t-test	X.Sam .	0.4204	0.7854	0.8441
	X.bar	0.9683	0.9718	0.9864
wilcox.test	X22	0.9586	0.224	0.2778
	Sample Par=1	0.9509	0.2099	0.2412
	Sample Par=0	0.6582	0.007248	0.5492

In bartlett. test we notice that the p. value is greater than 0.05, which means that the variance of the two groups is approximately equal at the level of significance 0.05, so we can say that there is homogeneity in all samples studied.

In the t- test, we find that the value is less than 0.05 which means the actual difference between the data is not equal to zero... so we choose the initial hypothesis according to Eq. 4, which mean that true difference in Mean is equal to zero. where: X0: Total, Y0: X1sample, X1: photo511MeV, Y1: X2 sample, X4: photo5111274MeV and Y4: X3 sample. for all the samples as figure 6.





Pearson's Product-Moment Correlation

The tables 5 and 6 shows the Pearson correlation coefficient test and the linear Regression test for the studied variables.

Tables 5. Pearson correlation

data	p.value	Correlation	Type correlation	95% percent confidence interval
Total.Net and Pho511 Net	0.936	-0.005715334	negative	-0.1443419 0.1331312
Pho511 Net and Pho124 7Nt	0.523	0.04441521	positive	-0.09491067 0.18203449
Total.Net and Pho124 7Nt	0.4472	0.05405186	positive	-0.08532906 0.19135784

We notice from Table 4. that the value of the correlation coefficient in the second and third groups was great than zero, which is a positive value, this means that there is a positive intermediate relationship between the variables, meaning that an increase in the net area of the photopeak of the sodium source is accompanied by an increase in the net area of the second photopeak. As for the first group, the correlation coefficient is negative and this means that there is a inverse relationship between the studied variables, We note the value of the correlation depends on the p. value, i.e. we consider as if there is no correlation between groups.

Figure 6. The figure shows the comparative of sampling mean



Table 6. Linear Regression test

data	p. value	Std. Error	Estimate	Min	Max	median	Residual	R-squared
Datta\$sample ~Total.Net	2.2e-16	2.755e-17	1.000e+00	-4.607e-14	5.674e-13	-2.990e-15	4.098e-14	1
Datta\$sample ~Pho511Net	0.936	1.624e-03	-1.306e-04	-107.00	967.73	-6.19	105.7	- 0.005018
Datta\$sample ~Pho1247Nt	0.4472	0.01553	0.01183	-130.88	956.91	-9.22	105.5	- 0.002114
Pho511Net~Total.Net	0.936	3.1103	-0.2501	- 12111.2	7310.3	-131.7	4626	- 0.005018
Pho1247Nt~Total.Net	0.4472	3.242e-01	2.469e-01	-1108.2	4584.6	-38.9	482.2	- 0.002114
Pho511Net~Pho1247Nt	0.5323	6.802e-01	4.255e-01	- 12109.8	7252.2	-181.8	4622	- 0.003068

Table 6. shows the simple linear regression of the studied samples, as the table contains the lowest error value, the highest value and the average error, as well as the value of the coefficient of determination and the residual value, which represents the total error value of the model, as the closer its value to zero, the better the quality of the model. The table also contains the p.value, which indicates the level of significance, as the results show that only the first group was less than the level of significance, while the rest of the groups were larger, and this means that the first group is

significant or has importance than the rest of the groups.

Multiple Linear Regression

A method similar to simple line regression, but used for more than variable together, according to Table 6. It is noticed from the table that the value of the adjusted R² is closer to the R² multiple in the first group of the multimeter because the sample size is relatively large.

Table 7. Multiple linear regression

data	p.value	Std. Error	Estimate	Min	Max	median	R-squared	p.value
sample	0.851	8.574e-14	-1.608e-14	-4.577e-14	5.660e-13	-2.980e-15	1	< 2.2e-16
Total.Net	<2e-16	2.771e-17	1.000e+00					
Pho511Net	0.802	6.327e-19	1.590e-19					
Pho1247Nt	0.588	6.071e-18	3.296e-18					

It was also found that the p.value, based on the Fisher statistic, is less than 0.05, and this indicates the existence of a variable and independent of one reason that has an effect on the samples. Thus,

Total.Net is the only significant group depending on the p. value.as figure 7.



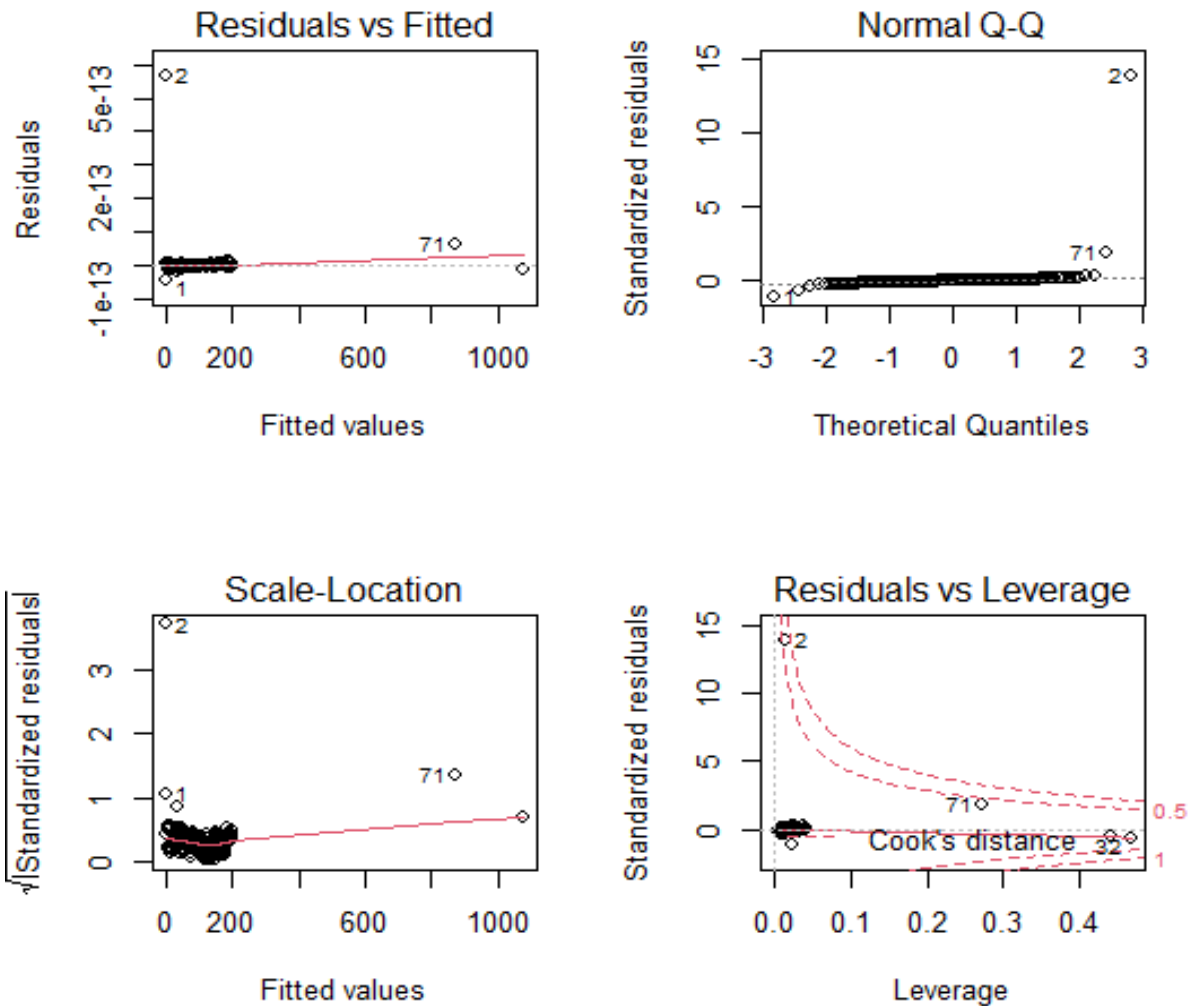


Figure 7. The figure shows Multiple linear regression

Figure 7 tests the conditions of linear regression. Regarding the first drawing (Residuals vs Fitted) it studies the validity of the existence of a linear relationship between sediments and the estimators. where we note that the relationship does not take a specific shape, which indicates the existence of a problem in the model to achieve this hypothesis. The second drawing (Normal Q-Q) shows the normal distribution of sediments. When data are collected around the straight line, it indicates a commitment of sediments to normal distribution. The third drawing (Scale Location) shows the homogeneity of sediment variation, that is, it shows the mechanism of data spreading, where the beginning and end of the drawing must be similar in shape, and when one end is different from the other, this indicates the heterogeneity of sediment variation. The fourth figure (Residuals vs Leverage) shows the presence of outliers, as the smaller the sample size

will lead to an imbalance in some of the requirements of multiple linear regression.

Conclusions

1. In Kolmogorov-Smirnov test found that the studied data in sampling random are not obey to normal distribution.
2. In Shapiro test, it was found that the net total area of the spectrum data was not subject to the normal distribution while the standard sample data were subject to a normal distribution depending on the p. value.
3. In Q.Q. Normal Test, we notice the points in Total Area is not applied along a straight line, while the data in photopeak's are applied along a straight line so the data is homogeneous.
4. By using linear regression test, we found that there is no strong statistical relationship between the studied random variables.
5. By using T-test, we found that the studied samples (Total Net Area, Photopeak Net



- Area) do not follow the normal distribution compared with the same random sampling standard that follow the normal distribution.
6. In Pearson correlation at the first and third groups was less than zero, which is a negative value, and this means that there is a weak inverse relationship between the studied variables. As for the second group, the correlation value was close to zero. positive, meaning that an increase in the net area of the first photopeak of the cobalt source is accompanied by an increase in the net area of the second photopeak.
 7. The correlation depends on the p.value as the second group has a p.value approximately equal to the value of level of significance 0.05, but in the first and third groups, the p.value was greater than 0.05, and this means not relying on the value of the correlation, i.e. we consider as if there is no correlation between first and third groups.

- Sitek A, Celler AM. Limitations of Poisson statistics in describing radioactive decay. *Physica Medica* 2015; 31(8): 1105-1107.
- Perales JL, Mendoza AG, Llanón CT, Pérez JA. A software for automatic calculation of radioactive decay and dispensation of radiopharmaceuticals. *Annul Congress of the Association of Nuclear Medicine* 2009.
- Kawano T, Talou P, Chadwick MB. Monte carlo simulation for statistical decay of compound nucleus. *In EPJ Web of Conferences, EDP Sciences* 2012; 21: 04001
- Ahmed SMS, Yahaya R, Radiman S. Clusterization probability in alpha-decay ^{212}Po nucleus within cluster-formation model; a new approach. *Romanian Reports in Physics* 2013; 65(4): 1281-1300.
- Norman M. *Probability and Statistics for Data Science Math + R + Data*. Taylor & Francis Group, LLC, USA 2020.
- Hadley W. *Advanced R*. Second Edition, USA 2019.
- Pedro JA. *Learn R As a Language*. Taylor & Francis Group, LLC, USA 2020.
- Köklü K. A quantum perspective model to genetic codes through various sciences. *NeuroQuantology* 2019; 17(3): 15-18.
- Pekdoğan S, Bozgün K. I can draw a scientist whom I imagined. *NeuroQuantology* 2019; 17(3): 1-8.

References

- Turner JE, Downing DJ, Bogard JS. *Statistical Methods in Radiation Physics*, USA 2012.
- Martin JE. *Physics for radiation protection*. Germany Wiley 2013: 270-279.
- Tobias NW, Antonino DP, Helge VK, Ulrik IU. Experimental evidence of quantum radiation reaction in aligned crystals. *Nature Communications* 2018; 9: 79.
- Khalid H.H. Al-Attayah, Ahmed M. Ali Al-Mehsenaay. *Study the Normal Statistical Distribution to Spectrum Radioactive Elements by Used Scintillation Detector Nal (TI)*. M.Sc., Dep. of Physics, College of Science, University of Babylon 2017.
- Nasrabadi MN, Sepiani M. Study of components and statistical reaction mechanism in simulation of nuclear process for optimized production of ^{64}Cu and ^{67}Ga medical radioisotopes using TALYS, EMPIRE and LISE++ nuclear reaction and evaporation codes. *In AIP Conference Proceedings* 2015; 1653: (1), 020076.
- Bakin RI, Kiselev AA, Shvedov AM, Shikin AV. Computational Errors in the Calculation of Long Radioactive Decay Chains. *Atomic Energy* 2018; 123(6): 406-411.
- Beard M, Uberseder E, Wiescher M. Statistical Model Calculations for (n, γ) Reactions. *In EPJ Web of Conferences* 2015; 93: 04001. EDP Sciences.
- Kaur DP. Statistical model calculations for the decay of ^{48}Ti + $^{136,140,142}\text{Ce}$ systems, *Nuclear Physics* 2017; 62(3): 542-543.
- Spectrum Techniques UCS-30_Manual, *The Universal Computer Spectrometer*, USA 2010.
- Gamma-Ray Spectroscopy Using Nal (TI)*, Experiment 3, ORTEC 2018.
- Neti, PV, Howell RW. Lognormal distribution of cellular uptake of radioactivity: statistical analysis of α -particle track autoradiography. *Journal of Nuclear Medicine* 2008; 49(6): 1009-1016.

