Study of Some Nuclear Properties of $^{170-178}$W Isotopes by Using Interacting Bosons Model-1

Salar H. Ibrahem[1*], Mohsin K. Al-Janaby[2]

Abstract

We looked into the structure of energy levels for some Tungsten isotopes W the (even-even) and electromagnetically probability transmission for it with the Model of Interacting Bosons type one (IBM-1) to evaluate the nuclear structure for Tungsten isotopes are investigated in this study ($^{170-178}$W). The data was gathered using an IBM software written in the Fortran programming language The values of the parameters in this computation show that the characteristics of tungsten isotopes fall between the γ_ unstable limit O(6) and the rotational limit SU(3). The energy and branching ratios demonstrate this. The computed results are in very excellent agreement with experimental data for the isotopes under investigation.

Key Words: Quadruple Momentum, Rotating Beams, Neutrons and Protons.

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Introduction

"The success in computing parity, spin, gamma, and beta decay in the ground levels, as well as the calculation of magnetic momentum gotten from the (singly particle) shell model, had already led to the evolution of geometrical models, but it has failed to determine and calculate quadruple momentum and rates of transition, especially between closed shells transition zones." A large number of nucleons to be moving in a mutual collective" this is assumed by this models. "A description properties of a set of nuclei is proficient to designate the connection of animated and rotating beams" (Hady, 2020).

IBM is one of these models that investigates the spectra of low-lying nuclear nuclei, assuming that the nucleus has an even atomic number and mass number, and is made up of a weak core with valence particles. Furthermore, the valence particles combine to create bosons with angular momentum L=2 (d-boson) and L=0 (s-boson) (Jaber and Muttaleb, 2019).

In the parity shell, these bosons are explained as int erconnected pairs of protons and interconnected pa irs of neutrons. This explanation limits the number of bosons, which is calculated by calculating the number of particle pairs (independently for neutrons and protons), the shell is less than half full if the shell is less than half filled, and the number of hole pairs if the shell is more than half filled. If protons' and neutrons' bosons were measured indistinguishably.

If the bosons of protons and neutrons are indisting uishable, the IBM is in its most basic form, which is known as IBM-1 (Muttaleb, 2012; Scholten, 1985; Casten, 1990).

The second version of IBM is IBM-2, which is a modification of IBM-1 that distinguishes between neutrons and protons (Casten and Warner, 1988; Kadhim, 2015), making it possible to distinguish between the wave functions of neutrons and protons.

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This model generates electric transitions and energy levels by adjusting a number of parameters in the model to fit the results with experimental data (Jaber and Muttaleb, 2019).

The nuclei form of $^{170-178}$W isotopes has been investigated analytically using IBM-1, and they have several infinite energy levels, with just the electric transition 2_1^+ to 0_1^+ only explored by T.K. Rz aca-Urban and Rz aca-Urban Nomura (8), Nomura In the IBM-1 model, the software set (IBM) version is used to approximate a set of parameters known as the Hamiltonian operator, as indicated in equation (1).

These parameters are handled as free parameters to get the best fit between theoretical estimated values and experimental data. The calculations allow for the efficient state that must occur in these parameters based on the nature of the neutron number decrease or increase for each nucleus.

The program set (IBMT) is used also to calculate the reducing electromagnetic transition prospects transitions in IBM-1.

Interacting Bosons Model-1(IBM-1)

The IBM-1 assumed that the Hamiltonian only has one- and two-body relationships, therefore it introduced the creation (s†, d†m) and annihilation (s, dm) operators, with m=0,±1,±2.

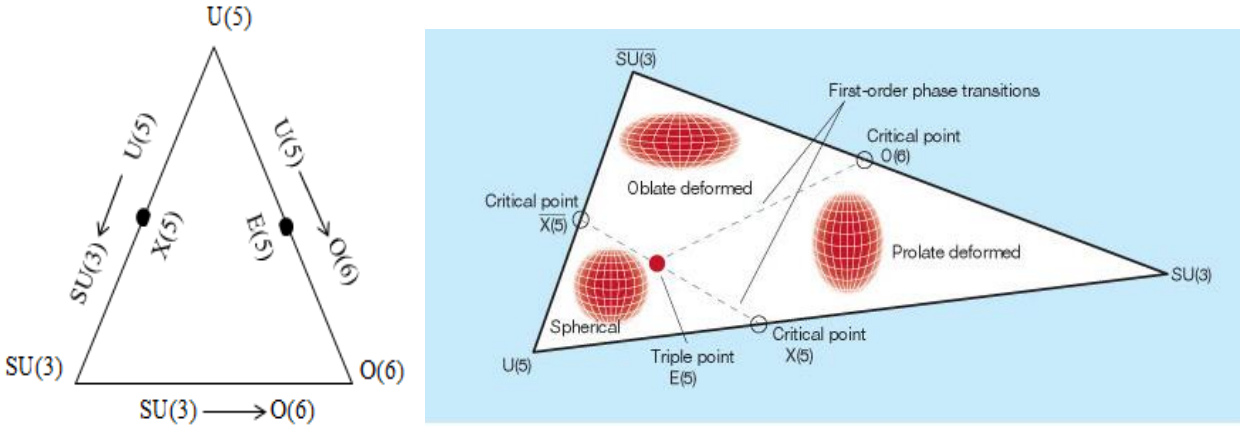
The IBM-1 supposed that the Hamiltonian comprises only one- body and two- body relations, thus introduced creation) and annihilation (s, d_m) operators, where m=0,±1,±2. The Hamiltonian with one –boson term in boson-boson interaction is the most general Hamiltonian (Nomura *et al*, 2017; Rząca-Urban *et al*, 2017; Iachello and Arima, 1987):

$$H = \in nd + a_0P.P + a_1L^2 + a_2Q^2 + a_3T_3^2 + a_4T_4^2 \quad (1)$$

where a_0,a_1,a_2, a_3, and a_4 are the parameters that we resolve in this model to achieve the output levels, and $\in$, L, P, T_3, Q and T_4 are the energy, angular momentum, pairing, octopole, quadruple, and hexadecapole operators, respectively.

Transitions Regions in IBM-1

Pairs of symmetries are an important aspect of symmetry notions since they serve as benchmarks and, as a result, transitional region limits. The nuclei concatenation in such a phase transitional area can be computed quite easily. Generally, their location along the appropriate leg of the symmetry triangle is determined by a variation of a single parameter. This parameter is usually calculated as the ratio of the coefficients in the Hamiltonians of the two symmetries conquering the triangle's apexes at the transition leg's termini. The Casten triangle displays the three dynamical symmetries as well as the transitional zones, as seen in Fig. (1).

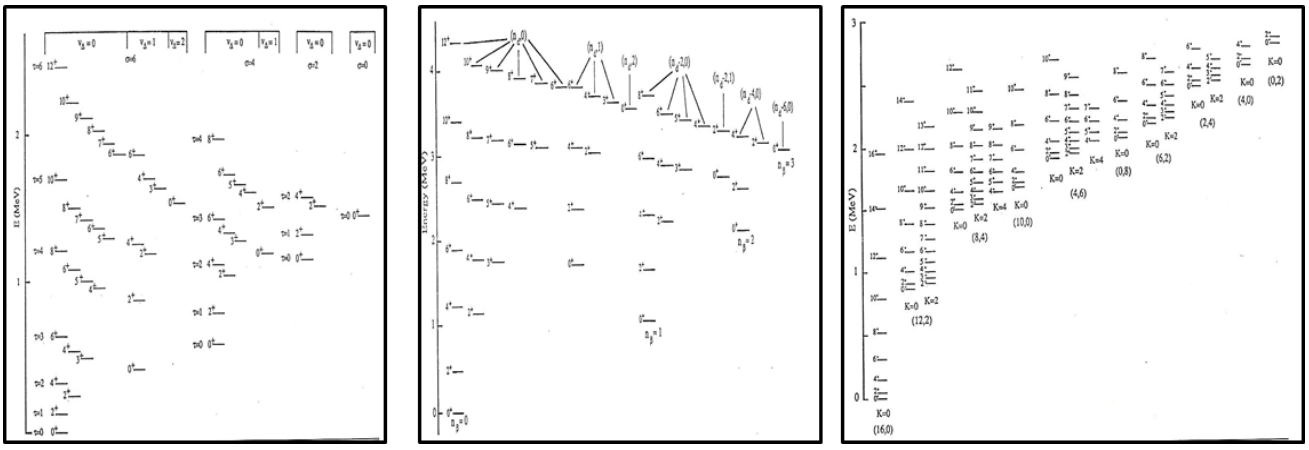


(a) (b)

Fig. 1. The Casten triangle (Warner, 2021)

Each peak in Fig. (b) represents an exact symmetry that is identical to one of the three forms shown. Transition points and their crucial symmetries, as well as first-order phase transitions, are defined. According to (Warner, 2021), "transition between a spherical nuclear form and a prolate or oblate distorted one. "There is a nuclear triple point that represents the second-order.

Fig.(2) represents a typical spectrum for the three dynamical symmetries, Fig.(2a) is O(6) typical spectrum symmetry, Fig.(2b) is SU(5) typical spectrum symmetry and Fig.(2c) is SU(3) typical spectrum symmetry.



a b c

Fig. 2. A typical spectrum with SU(3), SU(5) and O(6) symmetry (Jaber, 2020).

The limits previously discussed give a set of analytical solutions that can be easily tested, as the number of the nuclei that can be characterized by these limits is so few, because most of nuclei have common properties between these limits called transition region, which can be divided into four classes:

Class A: $\boldsymbol{U(5) \rightarrow SU(3)}$

The nuclei has properties between vibrational and rotational limit in the transition region, and the Hamiltonian operator is given as follows (Aboud, 1990):

$$\hat{H}^{I+II} = \varepsilon \hat{n}_d + a_1 \hat{L}.\hat{L} a_2 \hat{Q}.\hat{Q} \qquad (2)$$

The ratio $(\varepsilon \hat{n}_d / a_2)$ determines the properties of nuclei in this region. Thus, when the ratio gets higher, the properties become closer to U(5) limit, but when the ratio gets lower, the properties becomes closer to SU(3) limit.

Class B: $\boldsymbol{SU(3) \rightarrow O(6)}$

The nuclei has properties between rotational limit and γ_ unstable limit in the transition region and the Hamiltonian is (Tuli, 1998):

$$\hat{H}^{II+III} = a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} \qquad (3)$$

The ratio (a_0/a_2) determines the properties of nuclei in this region. Thus, when the ratio gets higher, the properties become closer to O(6) limit, but when the ratio gets lower, the properties becomes closer to SU(3) limit.

Class C: $\boldsymbol{U(5) \rightarrow O(6)}$

The nuclei has properties between vibrational limit and γ- unstable limit in the transition region and the Hamiltonian is (Tuli, 1998):

$$\hat{H}^{(I+III)} = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_3 \hat{T}_3.\hat{T}_3 \qquad (4)$$

Properties of this limit depend on the ratio $(\varepsilon \hat{n}_d / a_0)$.

Class D: $\boldsymbol{U(5) \rightarrow SU(3) \rightarrow O(6)}$

The nuclei of this class possesses the common properties between three limits and the Hamiltonian operator is given as follows (Tuli, 1998):

$$\hat{H}^{(I+II+III)} = \varepsilon \hat{n}_d + a_0 \hat{P}.\hat{P} + a_1 \hat{L}.\hat{L} + a_2 \hat{Q}.\hat{Q} + a_3 \hat{T}_3.\hat{T}_3 + a_4 \hat{T}_4.\hat{T}_4 \qquad (5)$$

Energy Ratios

Some ratios between the ground energy levels 8^+, 6^+ and 4^+ with the first excited state 2^+ to match the three limits as exposed in Table (1) (Jaber, 2020):
The analytical solvable dynamical symmetries SU(3) and O(6) with schematically descriptions – soft nuclei are used to describe axial symmetric rotations and spherical vibrators in IBM. [11]. The unstable region γ –soft, from the ratio between $E^+_{8_1}/E^+_{2_1}$, $E^+_{6_1}/E^+_{2_1}$ and $E^+_{4_1}/E^+_{2_1}$ as in Table(1).

Table 1. Typical energy levels ratios for each limits (Jaber, 2020)

Limit	$\frac{E_{4_1^+}}{E_{2_1^+}}$	$\frac{E_{6_1^+}}{E_{2_1^+}}$	$\frac{E_{8_1^+}}{E_{2_1^+}}$
U(5)	2	3	4
SU(3)	3.33	7	10
O(6)	2.5	4.5	7

Branching Ratio

Other important quantities show that the difference between the three limits are the branching ratio (Obiad, 2020):

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$$R=\frac{B(E2;4_1^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)},\ R'=\frac{B(E2;2_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)},\ R''=\frac{B(E2;0_2^+\to 2_1^+)}{B(E2;2_1^+\to 0_1^+)} \quad (6)$$

$$R=\begin{cases} U(5): & 2(N_b-1)/N_b & \to 2 \\ SU(3): & 10(N_b-1)(2N_b+5)/7N_b(2N_b+3) & \to 10/7 \\ O(6): & 10(N_b-1)(N_b+5)/7N_b(N_b+4) & \to 10/7 \end{cases} \quad (7)$$

$$R'=\begin{cases} U(5): & 2(N_b-1)/N_b & \to 2 \\ SU(3): & 0 & \\ O(6): & 10(N_b-1)(N_b+5)/7N_b(N_b+4) & \to 10/7 \end{cases} \quad (8)$$

$$R''=\begin{cases} U(5): 2(N_b-1)/N_b \to 2 \\ SU(3): 0 \\ O(6): 0 \end{cases} \quad (9)$$

Results and Discussion

The energy levels of isotopes were estimated using the (IBM) software in Fortran and the Hamiltonian as equation(1) with the values listed in Table (2). These parameters are gotten energy levels for ^{170}W, ^{172}W,^{174}W, ^{176}W and^{178}W and draw it in Fig.(2), (3), (4), (5) and (6) respectively, there is an excellent corresponding with the experimentally results [16-20] and it is so near from the typical spectrum with SU(3) symmetry (Fig.(2-c)). We can see that there are many uncertain energy levels or unknown parties in this investigation.

Table 2. The Hamiltonian's parameters in MeV units and the number of bosons (N).

The Isotopes	Boson Number	E	a_0	a_1	a_2	a_3	a_4	χ
^{170}W	11	0	0.0108	0.0155	-0.0276	0	0	-0.525
^{172}W	12	0	0.0102	0.0109	-0.0238	0	0	-1.097
^{174}W	13	0	0.0180	0.0114	-0.0238	0	0	-1.269
^{176}W	14	0	0.0220	0.0103	-0.0245	0	0	-1.255
^{178}W	15	0	0.0980	0.0123	-0.0168	0	0	-0.849

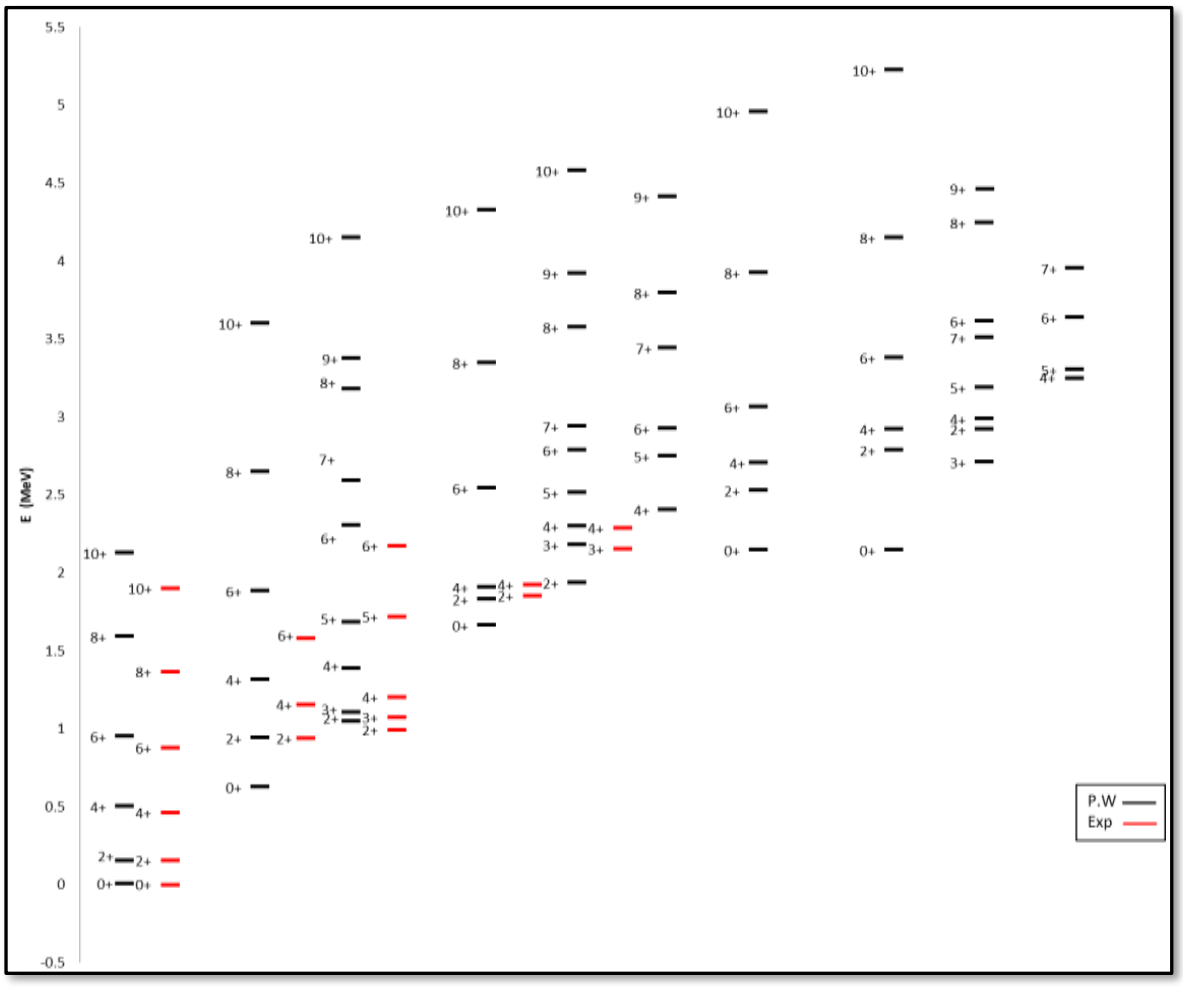

Fig. 3. Energy Levels for ^{170}W

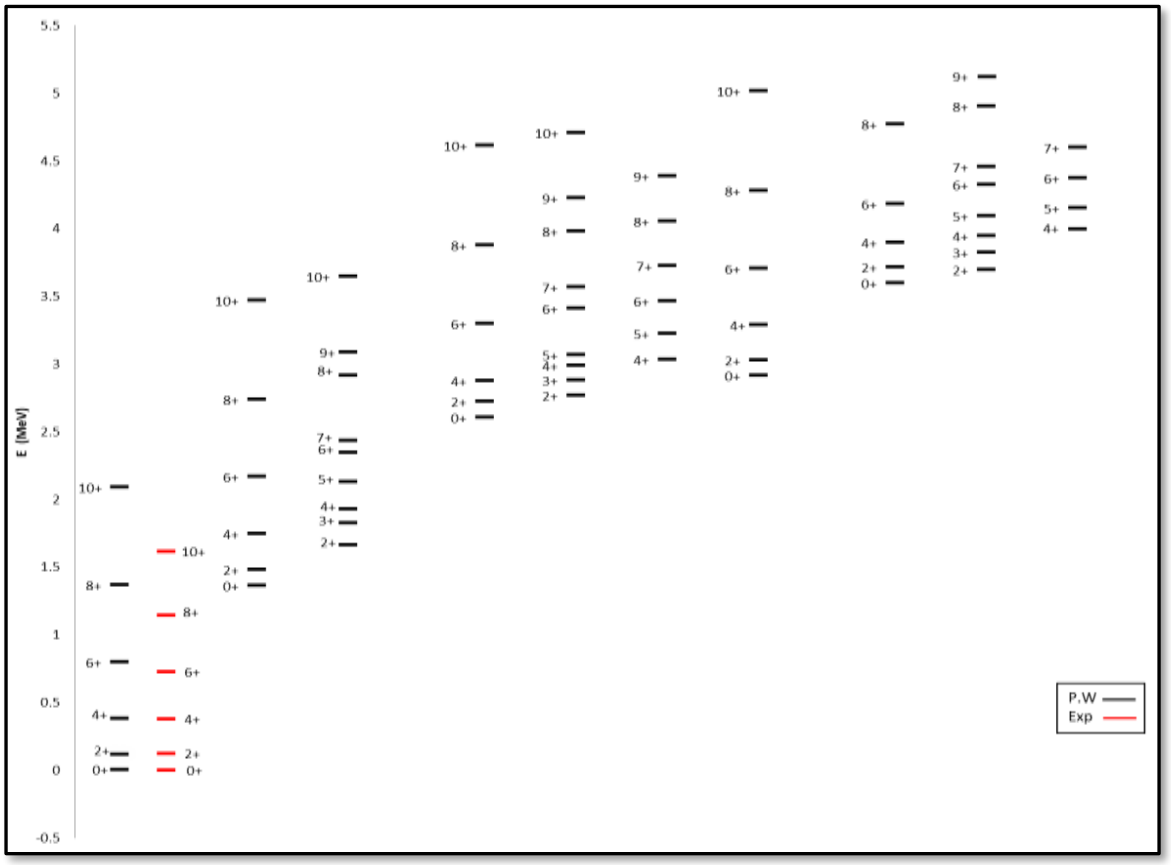

Fig. 4. Energy Levels for ^{172}W

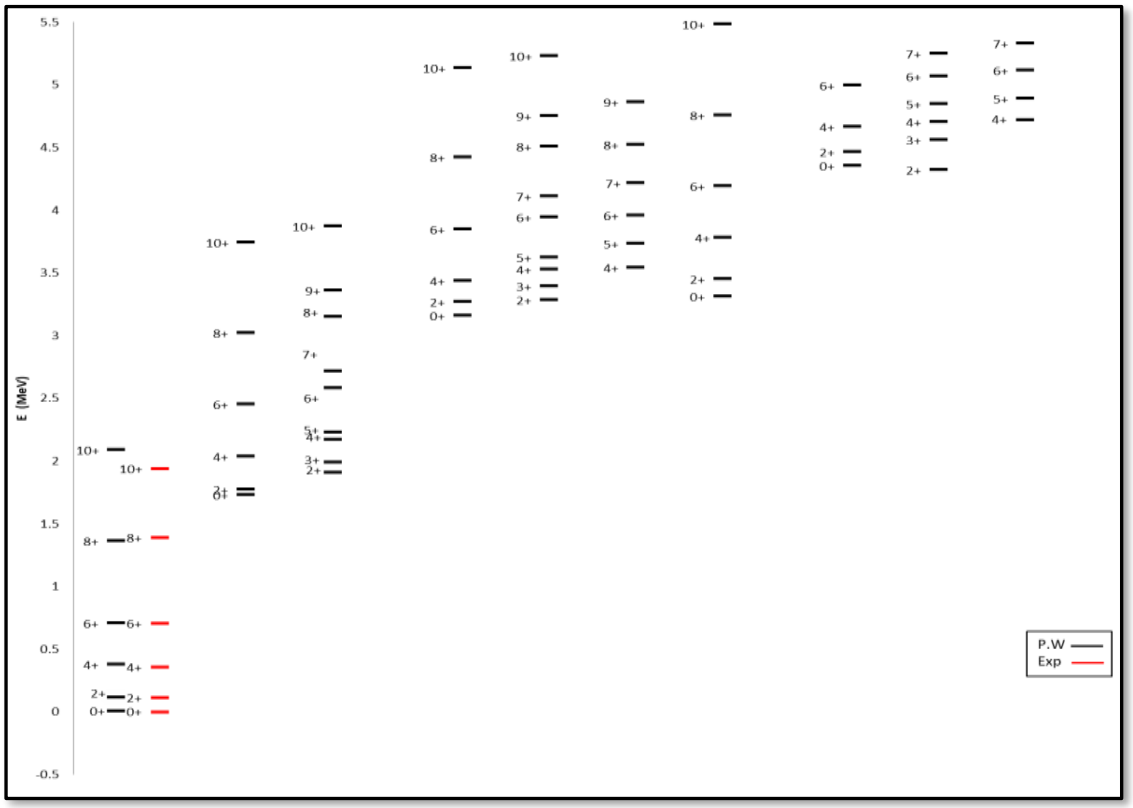

Fig. 5. Energy Levels for ^{174}W

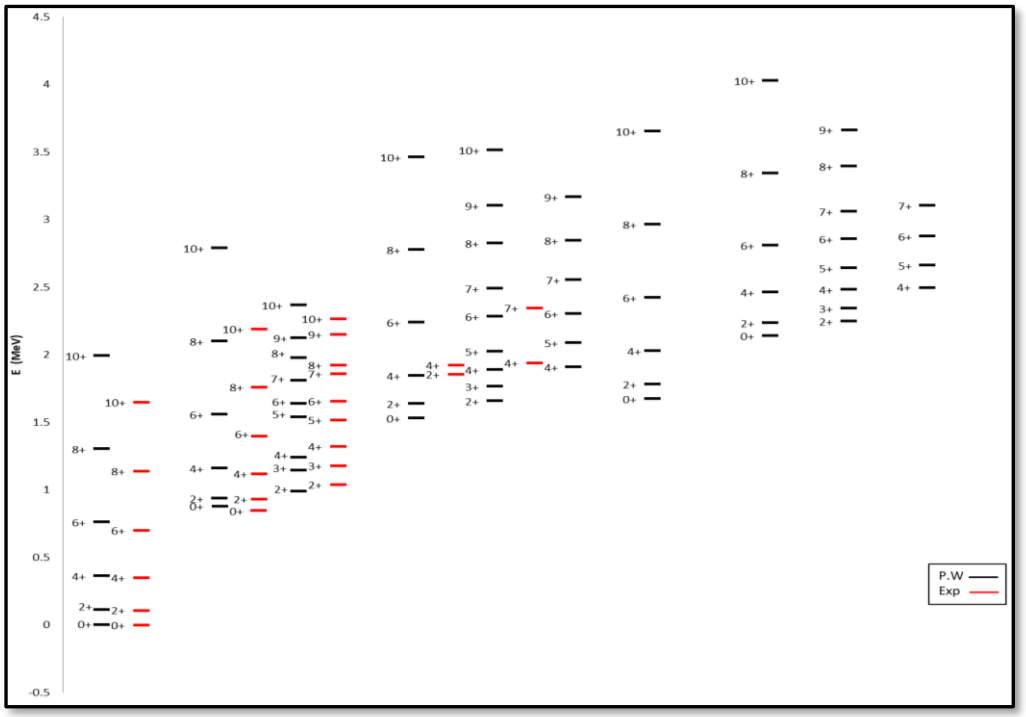

Fig. 6. Energy Levels For ^{176}W

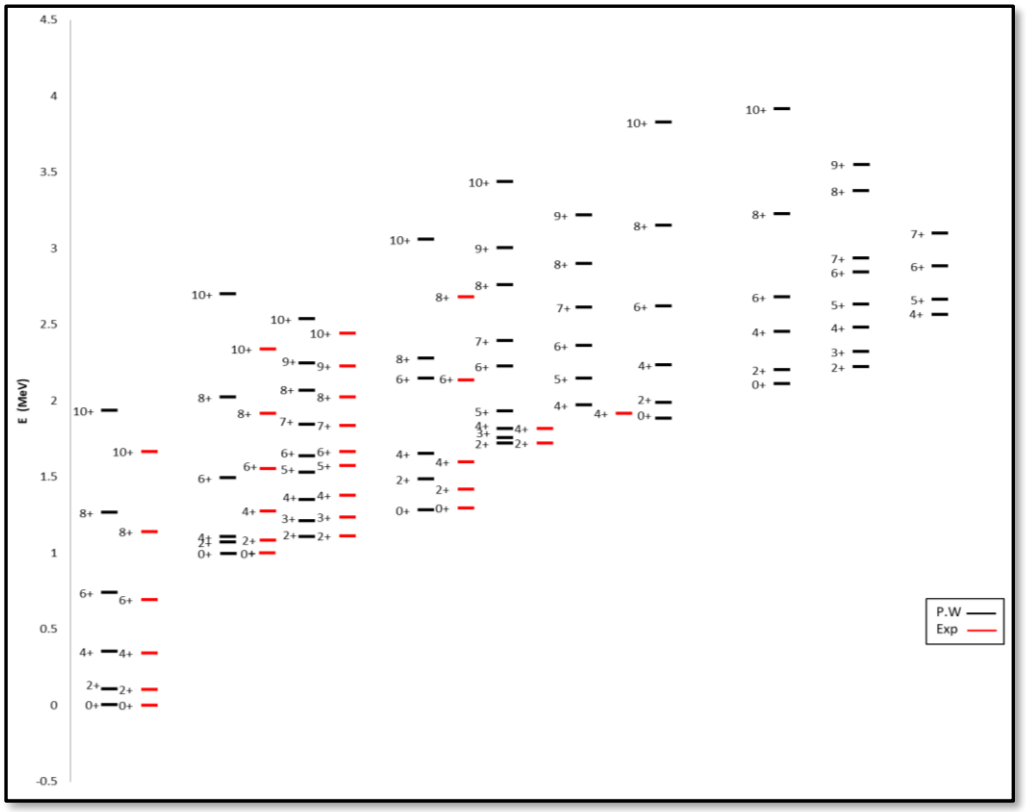

Fig. 7. Energy Levels For ^{178}W

For this current work and the experimental values in a very excellent matching with the theoretical values, we can conclude that $^{170-178}$W isotopes are located in the SU(3) - O(6) area, because the ratio of $E_{8_1}^+/E_{2_1}^+$, $E_{6_1}^+/E_{2_1}^+$ and $E_{4_1}^+/E_{2_1}^+$ for this present work, (Table(1)) this shown in Table(2) and in Figs. (8), (9) and (10).

Table 3. Experimental and present work energy levels ratios for W isotopes

Isotopes	^{170}W		^{172}W		^{174}W		^{176}W		^{178}W	
	Exp.	IBM-1	Exp.	IBM-1	Exp.	IBM-1	Exp.	IBM-1	Exp.	IBM-1
$E_{4_1}^+/E_{2_1}^+$	2.95	3.32	3.06	3.34	3.15	3.33	3.21	3.33	3.23	3.33
$E_{6_1}^+/E_{2_1}^+$	5.58	6.96	5.90	7.03	6.24	7.01	6.45	7.18	6.55	7.05
$E_{8_1}^+/E_{2_1}^+$	8.69	11.90	9.30	12.07	10.07	12.05	10.52	12.05	10.77	12.02

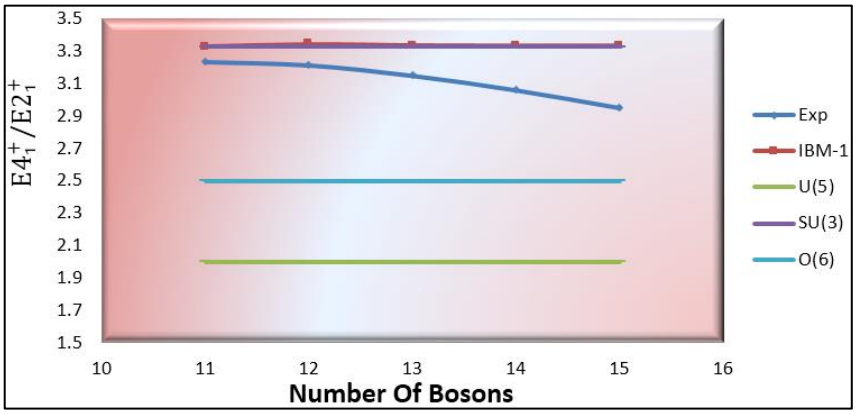

Fig. 8. $\mathbf{E4_1^+/E2_1^+}$ Ratios between experimental and the IBM-1 with standard ratios for $^{170-178}$W isotopes.

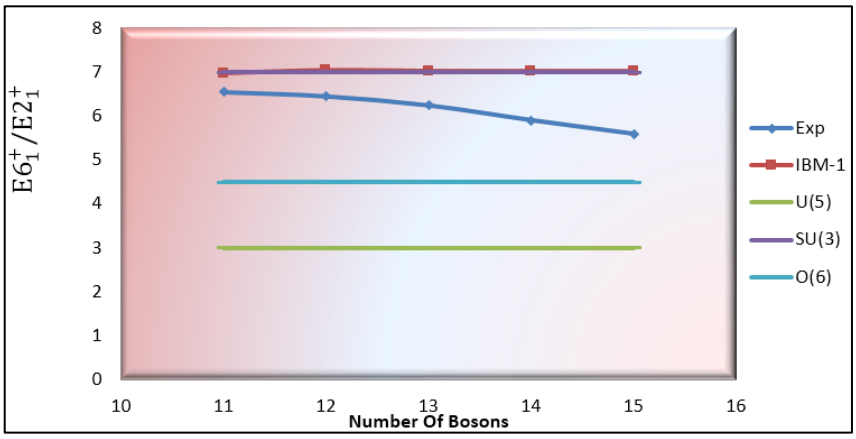

Fig. 9. $\mathbf{E6_1^+/E2_1^+}$ Ratios between experimental and the IBM-1 with standard ratios for $^{170-178}$W isotopes.

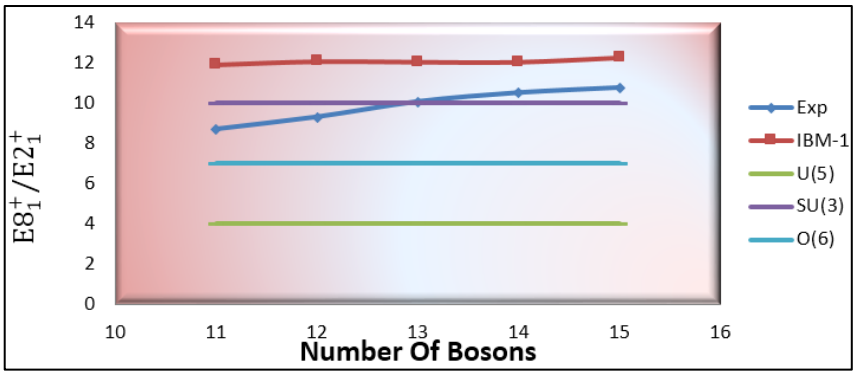

Fig. 10. $\mathbf{E8_1^+/E2_1^+}$ Ratios between experimental and the IBM-1 with standard ratios for $^{170-178}$W isotopes

The electric transition probabilities obtained from the IBMT algorithm were compared to experiment data in Table (4) for the transition from the initial level I to the final level (f), using E2DD and E2SD values. These transitions are significant because they allow us to calculate the probability of emission or gamma ray absorption at any energy. E2SD is equal effective charge(α_2), where (α_2) is dependent

on the limit of the region of isotopes in order to obtain the appropriate electric transition, and E2DD = $(5)^{1/2}$ β_2, where (β_2) is the parameter of deformation.

All the experimental data in this research are taken from (Baglin *et al*, 2018; Singh, 1995; Browne and Junde, 1999; Basunia, 2006; Achterberg *et al*, 2009).

Table 4. Experimental and present work energy levels ratios for W isotopes

The Isotopes	Boson Number	$B(E2;2_1^+ \rightarrow 0_1^+)$ e^2b^2 Exp.	$B(E2;2_1^+ \rightarrow 0_1^+)$ e^2b^2 IBM-1	E2SD	E2DD
^{170}W	11	0.712	0.710	0.107	-0.318
^{172}W	12	1.176	1.178	0.117	-0.348
^{174}W	13	0.684	0.665	0.080	-0.238
^{176}W	14	0.831	0.809	0.082	-0.244
^{178}W	15	0.838	0.838	0.082	-0.242

The branching ratio R, R' and R'' have been found in agreement both experimentally and theoretically (Table (4)) also with in consistence with their ideal corresponding limits as in equations (7),(8) and (9) depending on these equations and the results in Table (5) it have been found that $^{170-178}$ W isotopes are locating in SU(3) region.

Table (6) represents some of permissible electric transitions calculated in IBM-1model for **W isotope**.

Table 5. The branching ratio between two electric transitions for W Isotopes

The Branching Ratios	Isotopes									
	^{170}W		^{172}W		^{174}W		^{176}W		^{178}W	
	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1
$R=\frac{B(E2;4_1^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	1.44	1.33	1.43	1.36	1.74	1.37	-----	1.37	-----	1.37
$R'=\frac{B(E2;2_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	-----	0.09	-----	0.09	-----	0.08	-----	0.08	-----	0.07
$R''=\frac{B(E2;0_2^+ \rightarrow 2_1^+)}{B(E2;2_1^+ \rightarrow 0_1^+)}$	----	0.007	----	0.005	----	0.004	----	0.003	-----	0.004

Table 6. The electric transitions in (e^2b^2) with positive parity for W isotopes

The Isotopes	^{170}W		^{172}W		^{174}W		^{176}W		^{178}W	
$J_i^+ \Rightarrow J_f^+$	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1	Exp.	IBM -1
$2_1 \rightarrow 0_1$	0.712	0.710	1.173	1.178	0.684	0.665	0.831	0.809	0.838	0.838
$2_1 \rightarrow 0_2$	-----	0.026	-----	0.031	-----	0.014	-----	0.015	-----	0.019
$2_1 \rightarrow 0_3$	-----	0.007	-----	0.0002	-----	0.0001	-----	0.0001	-----	0.0006
$2_2 \rightarrow 0_1$	-----	0.048	-----	0.069	-----	0.036	-----	0.040	-----	0.038
$2_2 \rightarrow 0_2$	-----	0.0001	-----	0.007	-----	0.003	-----	0.003	-----	0.002
$2_2 \rightarrow 0_3$	-----	0.019	-----	0.026	-----	0.013	-----	0.015	-----	0.015
$2_3 \rightarrow 0_1$	-----	0.003	-----	0.002	-----	0.0008	-----	0.001	-----	0.001
$2_3 \rightarrow 0_2$	-----	0.412	-----	0.972	-----	0.560	-----	0.690	-----	0.713
$2_3 \rightarrow 0_3$	-----	0.039	-----	0.008	-----	0.005	-----	0.003	-----	0.002
$2_4 \rightarrow 0_1$	-----	0.0007	-----	0.00004	-----	0.00004	-----	0.00007	-----	0.00009
$2_4 \rightarrow 0_2$	-----	0.025	-----	0.002	-----	0.0003	-----	0.0003	-----	0.001

Finally, From the results had been calculated by IBMT program we found that there is a relation between the Q_{E2} and the number of bosons for the W isotopes, as in Table(6) all values is either greater or less than 0, which means that a deformation in nucleus for each isotope. this relation was represented in a diagrams in Figs. (11), (12) and (13). The deformation in nucleus is shown in Figs. (14-18).

Table 7. The quadrupole momentum Q_{E2} in (eb) for $^{170-178}$W isotopes

Isotopes	Bosons Number	Q_{E2} $2_1^+ \rightarrow 2_1^+$	Q_{E2} $2_2^+ \rightarrow 2_2^+$	Q_{E2} $2_3^+ \rightarrow 2_3^+$
^{170}W	11	-2.339	1.837	-1.933
^{172}W	12	-2.966	2.478	-2.717
^{174}W	13	-2.220	1.890	-2.046
^{176}W	14	-2.437	2.100	-2.259
^{178}W	15	-2.490	2.147	-2.310

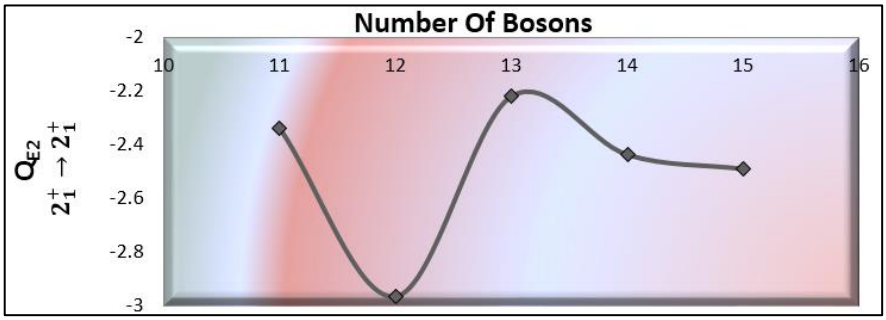

Fig. 11. The relation between quadrupole momentum $Q_{E2}(2_1^+ \rightarrow 2_1^+)$ and the number of bosons for $^{170-178}$W isotopes

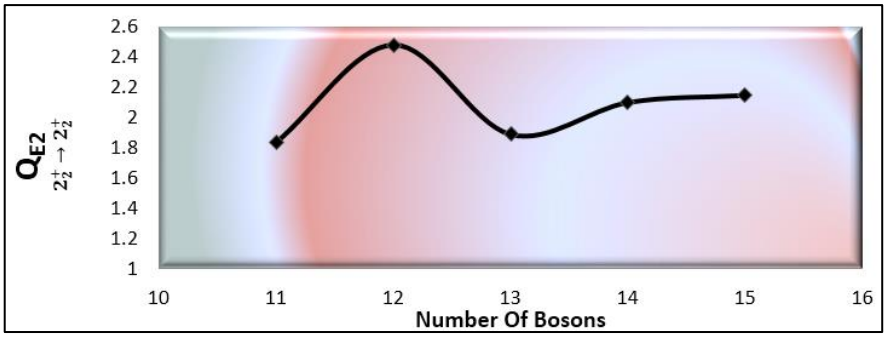

Fig. 12. The relation between quadrupole momentum $Q_{E2}(2_2^+ \rightarrow 2_2^+)$ and the number of bosons for $^{170-178}$W isotopes

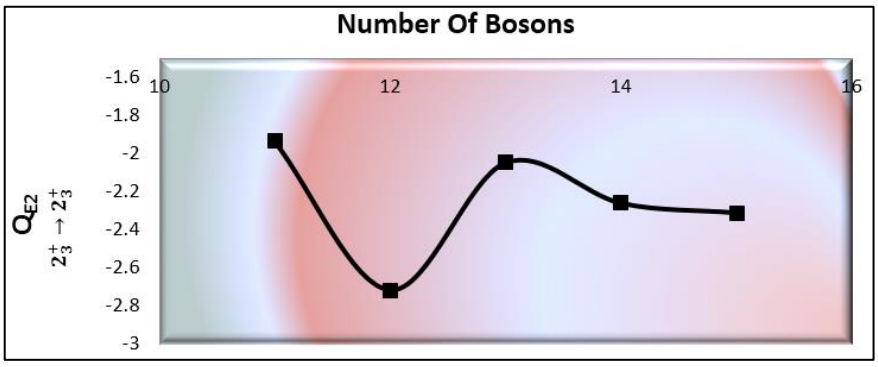

Fig. 13. The relation between quadrupole momentum $Q_{E2}(2_3^+ \rightarrow 2_3^+)$ and the number of bosons for $^{170-178}$W isotopes

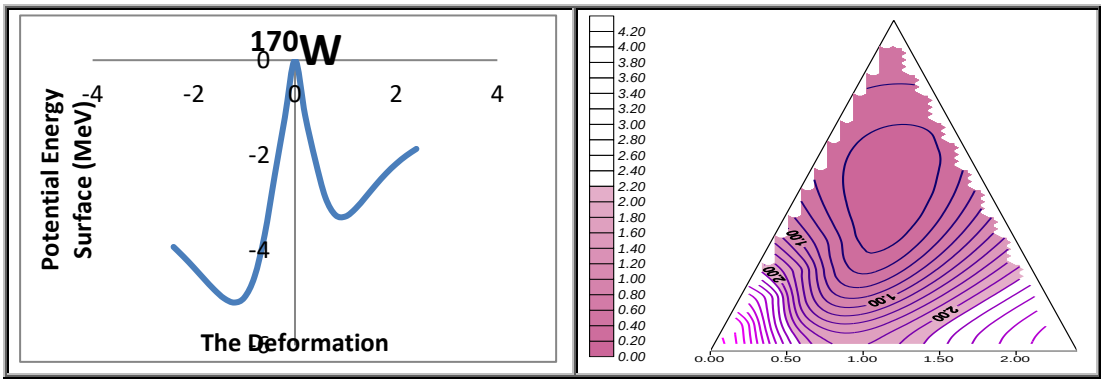

Fig. 14. Potential energy surface with the deformation for ^{170}W

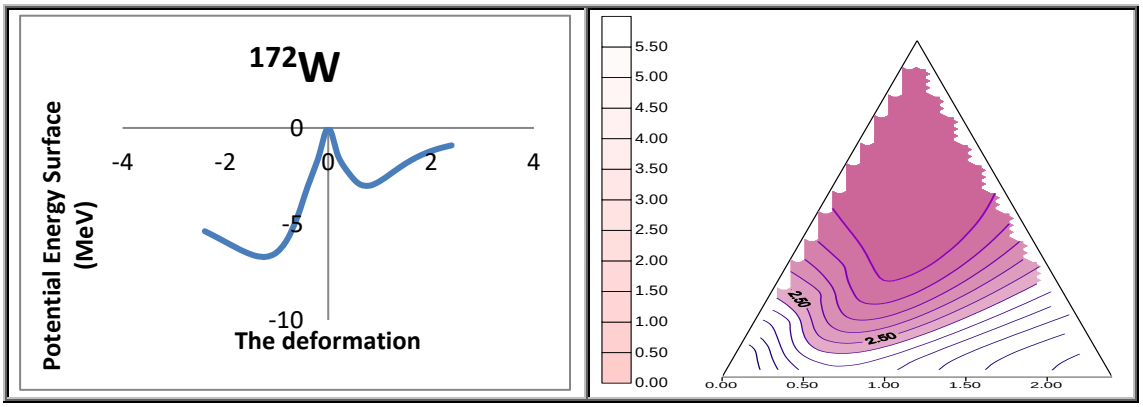

Fig. 15. Potential energy surface with the deformation for ^{172}W

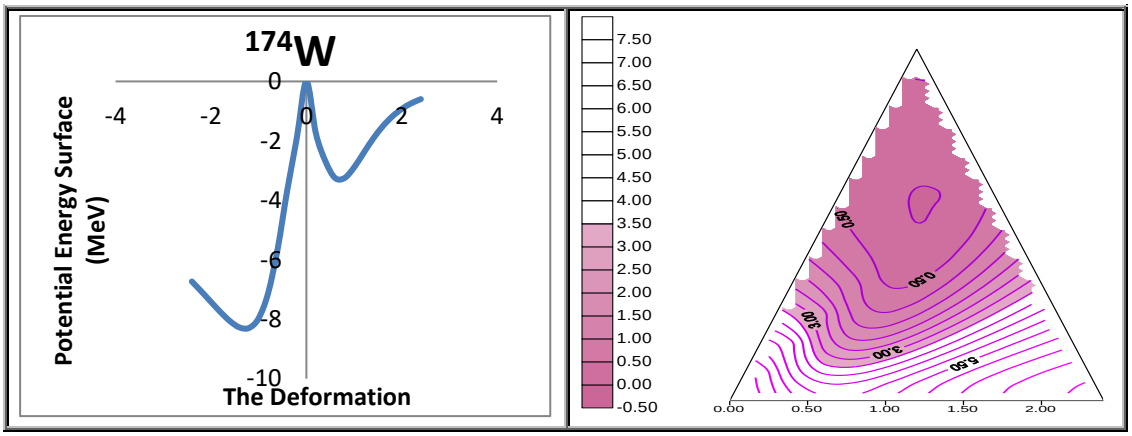

Fig. 16. Potential energy surface with the deformation for ^{174}W

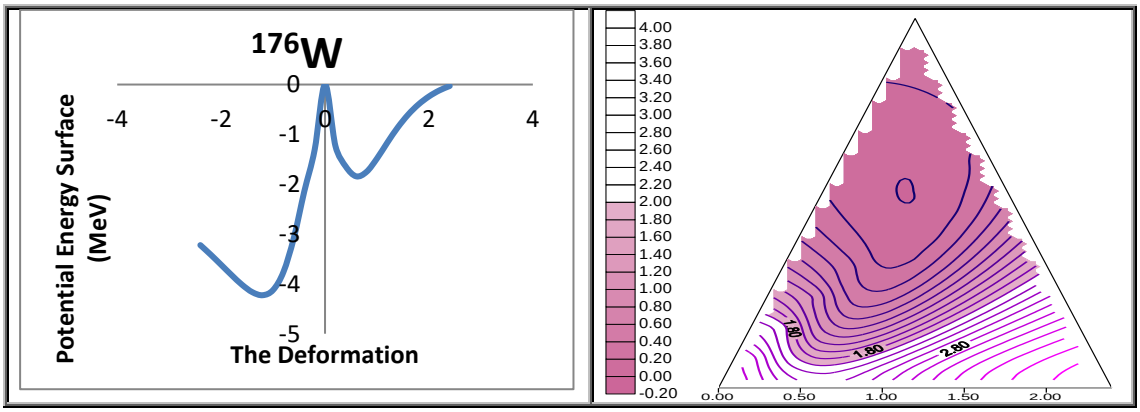

Fig. 17. Potential energy surface with the deformation for ^{176}W

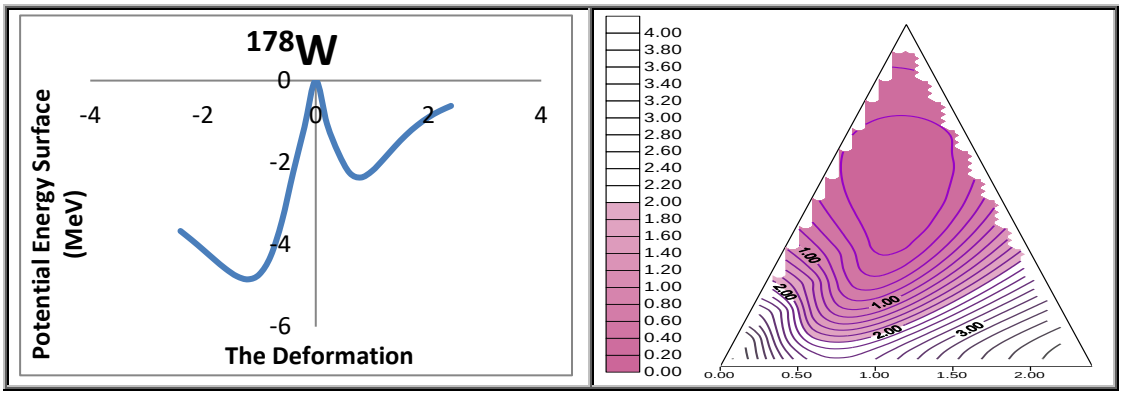

Fig. 18. Potential energy surface with the deformation for ^{178}W

Conclusions

1. Tungsten isotopes ($^{170-178}$W) have dynamical symmetry between SU(3) to O(6), because the ratio values are closely identical to the values in Tablet (1), and compared with Casten Triangle, which its Hamiltonian as eq.(5) and the branching ratios for it refers to the SU(3) region from the very good matching with typical values.
2. The angular momentum and parity of many high energy levels were confirmed with the experimental data. Unknown levels were been known with positive parity.
3. The branching ratios of the electric transitions are a good exam to know the limit of the isotope and emphasis it.
4. The calculated B(E2) values are in excellent agreement with the experimental data. Although there are numerous variations between them, such as the effect of deformation on the nuclei of these isotopes, there are many similarities.
5. All the isotopes of W in our study are in SU(3) limit.

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